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Is a Cosmological Substratum Compatible with Relativity?

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Abstract

A cosmological substratum for energy propagation is defined in terms of a hypothesis by McCrea. It has been shown that the assumption of such a substratum for a uniformly expanding universe provides a cosmological interpretation of Special Relativity, and leads further to a theory of gravitation in terms of a universal acceleration field. Following a critical discussion of the bases of General Relativity, it is suggested that the proposed substratum model and its consequences are also compatible with the General Relativistic approach, providing that this is applied in a manner which recognises the centrally directed character of gravitational fields, and hence employs harmonic coordinates as proposed by Fock. It is shown that Fock's procedure leads to results which are consistent with the assumption of a uniformly expanding cosmological substratum. Finally, it is suggested that the cosmological substratum concept is also implied by the formulation of the Robertson-Walker metric.

1. *Basis of a Cosmological Substratum*

Despite the many new and remarkable astronomical discoveries of recent years, the evidence continues to support the view that the observable universe can be considered as a homogeneous and expanding system of fundamental particles (the galaxies or clusters of such) governed by the Cosmological Principle. This Principle embodies the assumptions that the physical nature and behaviour of distant galaxies appears no different to those comparatively close, including our own, that the laws of nature as we know them appear to operate also in the distant parts of the universe, and finally that the general appearance of the universe including its expansion in all directions would appear to be the same from the viewpoint of any fundamental observer, that is an observer associated with any fundamental particle.

Astronomical observations are possible because radiation in all its forms travels vast distances through the universe; yet, to the best of our knowledge, light from distant receding galaxies reaches us with the same velocity (relative to our terrestrial reference frame) as that of light emanating from sources on the earth. This phenomenon, expressed by the statement 'Light never overtakes light', suggests strongly that the velocity of propagation of radiation is indeed independent of its source (as suggested also by de Sitter's observation of double stars), and that light signals originating from all parts of the universe have a common and unique velocity of propagation relative to any locality in the universe.

All this evidence is consistent with the hypothesis, first concretely expressed by McCrea (1962), that electromagnetic propagation has the velocity c relative to every fundamental observer in its path, that is, that light travels with constant velocity relative to a cosmological reference frame associated with the set of fundamental observers.

We note that this reference frame is observationally distinguishable from all other reference frames, for instance those associated with observers moving relatively to the fundamental observer in their vicinity; for the Doppler red-shift in radiation from distant galaxies appears isotropic only to the fundamental observers, that is in respect to what we may now call the fundamental reference frame. It is not isotropic in respect to any other reference frame, for example, as is well known, to one based on the earth or on the sun. The apparent absence of isotropy in the early observations of Doppler-shifts in the radiation from nearby galaxies led to considerable confusion until it was realised that correction needed to be made for the rotation of our galaxy and for the sun's (and earth's) proper velocity relative to the galactic centre.

The recent discovery by micro-wave astronomers of a uniform 3°K background radiation in our universe provides a second criterion for defining a cosmological reference frame, since the apparent temperature of this radiation should appear isotropic only with respect to the fundamental observers. Observations by Conklin (1969) are consistent with this viewpoint and with the independent evidence based on the Doppler red-shift observations. These astronomical observations therefore provide further support to the assumption that the nature of light propagation (and of all other forms of radiation) is indissolubly linked, as in McCrea's hypothesis, with the distribution and relative velocity of the fundamental particles of our universe; and hence that the universe manifests a fundamental reference frame which may be said to constitute a cosmological substratum for electromagnetic propagation.

2. *Special Relativity as a Cosmological Theory*

The existence of a fundamental reference frame for light propagation would appear to contradict Einstein's Light Principle and its consequences and also the null-evidence for such a reference frame implied by the Michelson–Morley type of observation. However, it has been shown elsewhere (e.g. Prokhovnik, 1967) that this conclusion is by no means necessary.

Einstein conceived his Light Principle as applying not only locally but on a universal basis, and the family of fundamental observers provides a perfect example of a set of observers, associated with local inertial reference frames, for whom the Laws of Nature take the same form. This was recognised by Milne (1948) who developed his Kinematic Relativity in terms of a uniformly expanding universe. Our model differs from his through the additional assumption of a specific mode of light propagation. It then follows directly and uniquely (e.g. Prokhovnik, 1967) that the set of fundamental observers are Lorentz-equivalent in conformity with their cosmological equivalence.

The assumption of McCrea's Hypothesis has many further implications; it implies isotropy of radiation with respect to any fundamental observer, but with respect to observers and bodies in motion relative to the fundamental reference frame the radiation will clearly no longer be isotropic. Instead such movement will be associated with a complex of anistropy effects as was recognised by Builder (1958). It may be shown (Prokhovnik. 1967) that these effects manifest themselves as relativistic phenomena resulting in the Lorentz equivalence of a uniformly moving observer with the fundamental observer in his vicinity. These same effects guarantee the constancy of the *measure* of the velocity of light and its apparent isotropy for all observers associated with inertial reference systems whether fundamental or subsidiary.† Thus in the context of our cosmological model, Special Relativity applies at two complementary levels resulting in the Lorentz equivalence of all inertial reference frames.

Relativistic phenomena will usually be due to the interaction of both cosmological and substratum effects. For example, the Doppler effect for light travelling relative to the substratum of our model manifests itself in two separate ways. The uniform expansion of the substratum leads directly to the cosmological red-shift effect, and for light sources and/or observers moving relative to the substratum there occurs also a substratum Doppler effect. In our context both of these effects assume the same Special Relativistic form, so that any combination of the two effects must also assume the same form. It is seen that the assumption of a uniformly expanding cosmological substratum is not only compatible with Special Relativity but provides a basis for the better understanding of the Theory and its consequences.

3. *Some Implications of a Uniformly Expanding Substratum*

The basis of our approach lies in considering the universe as an expanding system of fundamental particles relative to which the velocity of light propagation is uniquely determined. On this basis our cosmological model can be conceived as a velocity space and in particular, assuming the expansion is uniform, as a hyperbolic velocity space which has been described elsewhere (Prokhovnik, 1970a). However, because of the expansion effect, we can also consider (Prokhovnik, 1970b) our cosmological velocity space as varying with

† It may be noted that the Builder interpretation of Special Relativity, as a purely substratum theory having only local significance, would apply to *any* model of the universe in which McCrea's Hypothesis applies.

cosmic time–in effect, becoming weaker–and hence manifesting an acceleration field whose properties can also be considered as a function of the distribution of matter and of the decreasing density of matter and of radiation. By assuming that the strength of the field depends also on local variations of matter-density we obtain the usual form of the law of gravitation with G given by

$$G = \frac{3}{4\pi T^2 \rho_0} \tag{3.1}$$

where T is the reciprocal of the Hubble constant and ρ_0 the average density of matter in our model universe. This expression for G yields a numerical value of the right order of magnitude and resembles Milne's result (1948) which he obtains by treating the set of fundamental particles as a uniformly expanding hydrodynamic system. In our approach the gravitational law consequence also appears as a unique result of the uniform expansion, but the emphasis here is on the behaviour of energy propagation under such universal expansion conditions. In our model the expansion is accompanied by a weakening of the velocity space which controls the propagation of radiation, and the resulting acceleration field is essentially a manifestation of the variation (with respect to time) of the propagation behaviour of radiation in our universe. The recently observed existence of a universal background of black-body radiation which apparently partakes in the expansion of the universe gives credence and provides a physical basis for this interpretation.

By treating gravitation as an energy-behaviour phenomenon resulting from the interaction of a universal acceleration field and local variations of matter-density, it follows (Prokhovnik, 1970b) that the field associated with a body moving relative to the substratum will depend on the velocity and that a change in velocity must therefore require a change in energy. Hence the gravitational and inertial properties of a body are seen to have a common basis and a common dependence on its substratum velocity. The approach has many further practical and theoretical implications which are discussed elsewhere (Prokhovnik, 1968, 1972), but here we wish to consider two epistemological issues which are posed by the nature of our model.

4. *Problems of Validity and Status*

Our first consideration involves the generally accepted assumption that the universe is expanding. At present the only evidence for this, if such it is, is the systematic Doppler red-shift of radiation from distant galaxies and quasars; however the observed red-shifts could conceivably be due, wholly or partly, to causes other than recession of the sources, for example they may be due to photon-photon interaction involving the black-body background radiation (cf. Pecker *et al.*, 1972). In principle, the recession effect would be confirmed if, over a period of time, we were able to discern a systematic decrease in the observed intensities of all distant galaxies and, even so, reasons other than recession might be advanced for such a decrease.

Further our model depends on the expansion being strictly uniform; this condition is required for the Lorentz equivalence of the fundamental observers, and is also a prerequisite for the existence of a specific universal acceleration field associated with a hyperbolic velocity space. The astronomical evidence is not yet sufficiently precise to decide whether the apparent expansion involves an acceleration factor or not, but the assumption of zero acceleration is by no means inconsistent with observation (Godart, 1968). However, whether or not the expansion and its uniformity can ever be strictly observationally confirmed, we are suggesting that the operation of the Cosmological Principle, of Special Relativity, and of gravitation as a universal phenomenon depend directly, if not uniquely, on the assumption of a uniformly expanding universe in which light is propagated according to McCrea's hypothesis. Hence the validity of this assumption, in turn, receives direct support from cosmological and relativistic phenomena, as well as from other astronomical evidence that the universe is evolving, e.g. radio-sources observations, black-body background radiation, etc.

The chain of implications involved in the model also renders it falsifiable in a number of ways. Observations contradicting the Cosmological Principle, the expansion of our universe or merely the uniformity of this expansion would provide immediate grounds for refutation of the whole scheme. As with the Steady-State Theory, any model which attempts to describe the universe in terms of simple assumptions leaves itself highly vulnerable to observational refutation, and the wide and specific implications of our model make it particularly vulnerable in this regard.

A second and possibly even more serious criticism of our approach lies in its apparent independence of General Relativity. After all, Einstein with his General Theory produced a highly successful theory of gravitation and laid the basis of modern theoretical cosmology. It will therefore be instructive to examine the basis of Einstein's approach and to see whether our model has any relevance to the theoretical framework which he constructed. This may also shed further light on the theoretical validity of the cosmological model which we have postulated.

5. *The General Relativity Approach*

General Relativity is a theoretical pinnacle of man's striving to comprehend nature. It is 'general' in more than one sense; it provides a basis for describing physical laws in a manner which applies equally to all coordinate systems, and further this basis is compatible with many cosmological models of our universe. The generality of the Theory is achieved through the employment of Riemannian geometry and the tensor calculus which are themselves pinnacles in the mathematical generalisation of geometry and algebra.

Einstein based the Theory on three assumptions which can be expressed as follows:

I A Generalised Principle of Relativity which postulates that the laws of physics must be of such a nature that they apply to systems of reference in any kind of motion.

II A Principle of Equivalence which in its 'weak form' proposes the equivalence of gravitational and inertial mass, and in its 'strong form' the equivalence of a gravitational field with a uniformly accelerated reference frame.

III A Principle of Covariance which requires that the general laws of nature are to be expressed by equations which hold good for all systems of coordinates, that is, are covariant with respect to any substitutions whatever.

As a first step towards providing a mathematical expression of these principles, Einstein required that the invariance of the Special Relativistic metric

$$\begin{aligned} ds^2 &= dt^2 - \frac{1}{c^2}(dx^2 + dy^2 + dz^2) \\ &= dx_1^2 - dx_2^2 - dx_3^2 - dx_4^2 \end{aligned}$$

be generalised such that, in tensor form,

$$ds^2 = \sum_{i,\, k=1}^{4} g_{ik}\, dx^i\, dx^k \tag{5.1}$$

should hold as an invariant with respect to all observers and all systems of coordinates. The elements of the metric tensor, g_{ik}, which may be functions of the coordinates at a point in space-time, describe geometrically the field properties at the point, and hence may be employed to describe a gravitational field. Einstein's goal was to find a formulation for the gravitational field satisfying the invariance condition (5.1) as well as the Covariance Principle. This he achieved through his gravitational field equations

$$R_{ik} - \tfrac{1}{2} g_{ik} R = -\kappa T_{ik} \tag{5.2}$$

where R_{ik} is the curvature tensor, g_{ik} is the metric tensor, R is the scalar curvature, T_{ik} is the mass-energy tensor, and $\kappa = 8\pi G/c^2$, G being the gravitational constant. These equations are the General Relativistic analogue of Poisson's equation for a gravitational field and satisfy the covariance condition of taking the same form for all coordinate systems. They also embody some fundamental assumptions and results of Special Relativity, in particular, the mass-energy equivalence law and the Light Principle generalised such that it applies to all forms of energy including gravitational energy.

6. *General Relativity Versus a Unique Universe*

General Relativity is justifiably a source of delight and pride for mathematicians; its generality is equated to its profundity of description, and its initial success in explaining and predicting a number of astronomical phenomena suggested that its generality could also be equated with universal validity. This belief stimulated the efforts for yet a further generalisation which might then provide a unified description of all physical phenomena—a unified field theory.

However, for physicists and astronomers, General Relativity did not prove as useful for solving problems or comprehending nature as had been hoped. Its generality and sophisticated mathematical formulation made application of the Theory difficult. The determination of exact solutions of the sets of partial differential equations provided by the Theory posed a prohibitive task, and approximate solutions were often merely equivalent to those obtained through a simpler approach (cf. Bondi, 1961). For cosmologists, there is the further difficulty that the Theory does not define or imply a particular model of the universe, so that the boundary and initial conditions (and consequently a solution) for problems with a cosmological setting may depend on arbitrary criteria.

Perhaps even more serious has been the recent critical re-evaluation of Einstein's main assumptions. His Principle III of Covariance was seriously weakened when it was realised (North, 1965) that practically any physical law can be expressed in a covariant form which need not be unique.

His Principle II of Equivalence, in its strong form, is mathematically very useful but involves an approximation in its postulation; it substitutes an accelerating reference frame for a centrally directed acceleration field. It is there fore only a valid substitution for a single direction of the field and then only if we can consider the acceleration as uniform with respect to position and time—which is not the case in a gravitational field even for a given direction. Thus Einstein's elevator thought-experiment (Einstein, 1920) suggests that a man inside a uniformly accelerated closed box could consider that he was stationary in a gravitational field; this certainly points to equivalent aspects of the two situations (accelerated frame and gravitational field), but the equivalence is not exact. If a number of bodies were suspended by long strings from the 'top' of the box, then they would all point to the centre of gravity if a gravitational field were involved; however this would not be the case—the hanging strings would now be strictly parallel—if the box were merely uniformly accelerated in a gravity-free region. It might be argued that this distinction between the two situations is trivial, however, it is certainly a distinction in principle and perhaps even in practice given a big enough box, long enough strings and modern electronic techniques of measurement.

The importance of the distinction between the two situations depends on the context of reference. In dealing with local dynamical systems, even those as large as the solar system, the application of the Equivalence Principle is justifiable and useful. However, as Fock (1959) observes, 'the equivalence of fields of acceleration and gravitation is strictly local', and in the cosmological context the two fields are no longer equivalent in any sense. Hence from the viewpoint of cosmology the strong form of the Principle II has a very limited validity and provides no help in describing the structure of the gravitational field and its relationship, if any, to the nature of the universe.

The weak form of the Principle II, the equivalence of gravitational and inertial mass, is postulated by Einstein on empirical grounds. The problem of elucidating how and why Mach's principle applies in a manner which produces this equivalence is clearly a cosmological issue, related possibly to the nature of the gravitational field. Einstein did not attempt to deal with this problem.

Finally, the Principle I of General Relativity is considerably weakened by the astronomical evidence which enables us to now define and distinguish a specific fundamental reference frame, not merely in a vague manner in terms of 'the fixed stars', but quantitatively in terms of a set of mutually receding fundamental particles and an associated black-body radiation background permeating the observable universe. Fock (1959) insists that this Principle has no deep significance; it does not reflect a uniformity of space-time as does the restricted Principle of Relativity, and merely leads to an indeterminateness of Einstein's gravitational equations. This generality is then a negative virtue according to Fock and he proposes instead the determination of Einstein's equations by the employment of a preferred set of coordinates appropriate to gravitational fields.

What is left then? Why does the Theory still 'work'? It works and is an improvement on Newton's approach because Einstein employs implicitly two assumptions which can be considered as consequences of Special Relativity. These are that the effective (gravitational and inertial) mass of a body is affected by its velocity, hence he employs a mass-energy tensor; and that any modifications in a gravitational field are propagated with the limiting velocity of light, hence recognising that a gravitational field manifests a form of energy. This latter consequence implies an inverse square law (for the dispersion of energy from a static source), and of course Einstein simply assumed the Newtonian value for the gravitational constant–he had no basis from which to derive it. Surdin (1962) has shown that the assumption of these two special relativistic consequences is sufficient to deduce the same results for the precession of the orbit of Mercury and for the deflection of light in a gravitational field as obtained from the General Relativistic equations. Nevertheless Einstein's formulation (5.2) remains the most comprehensive and powerful tool for describing gravitational fields, and it will be seen that this formulation is by no means inconsistent with the concept of a cosmological substratum as suggested by our present view of the universe.

7. *The Case for a Preferred System of Coordinates*

Einstein based his Special Theory on the equivalence of all inertial systems, even with respect to light propagation, and so appeared to render an aether concept entirely redundant; yet, after he had developed his General Theory, Einstein took a more positive attitude to the properties of space. He saw the universe in terms of interactions, between particles of matter and fields, with the latter not necessarily subsidiary to the former but perhaps even primary, whereby matter then appears as singularity manifestations of the field. Hence it was empty space which now became the redundant concept; the space of the universe was endowed with properties associated with (or perhaps which even determined) the distribution and behaviour of matter; it was the carrier of energy and the means of interactions between particles and Einstein thought of it as a new universal aether bearing, however, little relationship to its nineteenth-century predecessors. Einstein frequently commented and speculated on this matter, and, in particular, provided an elaboration of his viewpoint in 'Über den Aether' (1924).

Strangely enough, these views were almost completely ignored by Einstein's contemporaries, perhaps because he never gave them specific and quantitative expression in terms of a particular model of the universe. His approach was subsequently developed by Builder and, from an entirely different direction, by Fock. Builder (1958) saw the existence of absolute relativistic effects (e.g. time-dilatation) as demanding logically the recognition of an absolute basis for motion. Leaning heavily on Einstein's published views, he therefore postulated a universal substratum (or aether) relative to which the propagation of light is isotropic, and conceived the relativistic consequences as anisotropy effects on moving bodies and on the measurements of moving observers. These effects lead in turn to the observational equivalence of inertial systems, so that Builder's approach provides a complete neo-Lorentzian interpretation of Special Relativity. Builder's substratum subsequently assumed a firm cosmological basis from the hypothesis of McCrea (1962) that light propagates relative to the set of fundamental observers.

Clearly, Einstein had hesitated in taking the step of making his universal aether synonymous with a basic reference frame for light propagation; such a step appeared precluded by his own Special Theory, and it required the further step taken by Builder to reconcile Einstein's aether and relativity convictions by demonstrating the anisotropy consequences of a substratum theory.

8. *A Particular Formulation of General Relativity*

Fock's approach (1959) involved quite different considerations. Since gravitational fields are characteristically centrally directed, their mathematical representation should reflect this property; hence he proposed that any solution for the potential ψ of Einstein's gravitational equations (5.2) should also satisfy harmonic coordinate conditions, viz. d'Alembert's equations, $\Box\psi = 0$. The solution depends also on the boundary conditions associated with the field. For an isolated system of masses Fock specifies such conditions by considering the associated field as imbedded in uniform Galilean space. Hence for such a system the harmonic coordinates constitute a preferred reference frame apart from a Lorentz transformation.

The boundary conditions assume a more subtle form if the apparent mass-energy structure of the universe at large is taken into consideration, or alternatively if Einstein's gravitational equations are considered in the context of a non-empty universe which can be taken as homogeneous and isotropic. Following on the work of de Sitter, both Lemaître and Robertson were able to show that such a universe must be expanding. Fock invokes the further result of Friedmann (1922) that the expansion is uniform and associated with a hyperbolic velocity space. This expanding 'Friedmann-Lobachevsky space' provides an ideal setting for the operation of Special Relativity, as shown independently by Robb (1936), and also for the employment of harmonic coordinates to describe the gravitational fields imbedded in this space. Fock therefore suggests that Friedmann-Lobachevsky space provides a better description of the boundary conditions for a gravitational field than does Galilean space, particularly for regions of cosmological dimensions.

Fock surmises that, apart from the special role of harmonic coordinate systems, Friedmann-Lobachevsky space may be associated with a preferred system of coordinates. Although he is not prepared to fully commit himself on this matter, he employs a uniformly expanding system of galaxies as the basis of a reference frame for light propagation to demonstrate the Doppler red-shift consequence. He then further deduces a cosmologically based formula for the gravitational 'constant' which is almost identical with our relationship (3.1). This is, of course, by no means fortuitous, since Fock's procedure leads to the assumptions of our cosmological model.

The difference between the two approaches lies only in their different starting points. Fock postulates the existence of centrally directed gravitational fields with boundary conditions and deduces that such fields operate in a uniformly expanding universe whose velocity space is hyperbolic as required by Special Relativity. Conversely, the assumption that light propagates relative to a uniformly expanding universe provides a sufficient basis to deduce the cosmological and substratum Doppler effects, the operation of Special Relativity on two complementary levels one of which involves the hyperbolic-velocity space of Robb and Friedmann, and the existence of a cosmological acceleration field underlying centrally directed gravitational fields as required by Fock. The two approaches are therefore essentially complementary and so provide mutual support for their respective assumptions whose only common feature is their separate postulation of a unique universe.

Whether or not these models are complementary and valid representations of our universe, their reconciliation has important theoretical implications. It demonstrates that a cosmological substratum is consistent with Einstein's gravitational equations providing these are restricted to the description of a particular universe, and conversely that the application of General Relativity to describe gravitational phenomena most generally does not preclude the assumption of a cosmological substratum. Indeed the two approaches can be considered to reinforce one another and to broaden our understanding of each of them; they vindicate Einstein's gropings for a universal substratum—'a continuum possessing physical properties' (Einstein, 1924)—with local and cosmological features determined by the distribution of matter.

9. *The Robertson-Walker Metric for a Uniformly Expanding Universe*

It was shown independently by H. P. Robertson and A. G. Walker that Einstein's generalised metric representation (5.1) could be employed to describe a model of a non-static universe which conformed to the Cosmological Principle. This representation incorporates a scale-factor, $R(t)$, corresponding to the nature of the expansion assumed, and also a parameter k corresponding to the assumed space-curvature of the model. In view of the assumed isotropy and homogeneity of the model, k is taken as constant on the cosmological scale.

The reference frame is based on the set of fundamental observers constituting the model, and clearly any one of these, say F_0, can be taken as the origin of the frame. In terms of spherical polar space coordinates, the Robertson-Walker metric is then given by

$$ds^2 = dt^2 - \frac{1}{c^2}\frac{R^2(t)}{(1+\frac{1}{4}kr^2)^2}\,(d\mathbf{r}^2 + \mathbf{r}^2\,d\boldsymbol{\theta}^2 + \mathbf{r}^2\sin^2\boldsymbol{\theta}d\boldsymbol{\phi}^2) \qquad (9.1)$$

where $\mathbf{r}, \boldsymbol{\theta}, \boldsymbol{\phi}$ are the fixed co-moving coordinates of any fundamental particle or observer relative to F_0 at the origin of the system, and t its cosmic time coordinate.

The curvature parameter k may take values of 0, +1 or -1 depending on whether the geometry of the model's 3-space is assumed to be Euclidean, spherical or hyperbolic respectively. Since this metric obtains equally for all fundamental observers, the interval ds is invariant with respect to all such observers in conformity with General Relativity.

For our uniformly expanding model described earlier, we may take $R(t) = t$ and $k = 0$. Further the fixed coordinate $\mathbf{r}$ associated with any given fundamental particle or observer can be taken, in our context, as equivalent to its constant recession velocity w relative to F_0 at the origin of the system. We note that $\mathbf{r}$ or w, though fixed with respect to a given fundamental particle, takes different values for a set of fundamental particles along (say) the path of a light ray emanating radially from F_0, and for such a path we can take $d\boldsymbol{\theta} = d\boldsymbol{\phi} = 0$. We can also define another radial distance coordinate r such that

$$r = wt = \mathbf{r}t \qquad (9.2)$$

so that r is the time-dependent distance of a given fundamental particle from F_0, and (9.2) can be considered as an expression of Hubble's Law.

Consider now the path of a light ray whose source can be taken as the origin of our reference system so that the path is radial and constitutes a geodesic with $ds = 0$ and $d\boldsymbol{\theta} = d\boldsymbol{\phi} = 0$ also. Remembering that for our model we are also taking $R(t) = t$ and $k = 0$, (9.1) then becomes

$$0 = c^2\,dt^2 - t^2\,d\mathbf{r}^2, \qquad (9.3)$$

which describes the radial path of the light ray. The solution of (9.3) is

$$\mathbf{r} = w = c\ln\frac{t}{t_0} \qquad (9.4)$$

where t_0 is the cosmic time epoch of the transmission of the ray at the origin. It follows that the distance r travelled by the light ray is given by

$$r = \mathrm{r}t = ct \ln \frac{t}{t_0} \tag{9.5}$$

The results (9.4) and (9.5) are identical to those which follow directly from the application of McCrea's Hypothesis to a uniformly expanding universe (Prokhovnik, 1967), so that this hypothesis and its consequences are fully compatible with the usual metric representation of such a universe.

The employment of (9.1) for a uniformly expanding universe has a further interesting consequence. Putting $\mathrm{r}t = r$ so that $t\, d\mathrm{r} = dr$, etc., transforms (9.1) to the metric of Special Relativity, viz.

$$\begin{aligned} ds^2 &= dt^2 - \frac{1}{c^2}(dr^2 + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2) \\ &= dt^2 - \frac{1}{c^2}(dx^2 + dy^2 + dz^2) \end{aligned}$$

in terms of cartesian coordinates.

Hence it also follows directly from the Robertson-Walker metric that observers are Lorentz-equivalent providing their mutual recession is strictly uniform. The result is unique since no other positive expression for $R(t)$ leads to it; the result is also unique in its manifestation of the cosmological equivalence of fundamental observers.

Finally we note that the form of the Robertson-Walker metric not only defines a unique cosmological reference frame, based on a set of fundamental observers, but it also implies that light propagation takes place with velocity c relative to this reference frame and hence to the set of these observers. Thus the existence of a cosmological substratum is by no means incompatible with any aspect of Relativity; indeed it is implied by the absolute effects of Special Relativity, by the restriction of General Relativity to a universe with centrally directed gravitational fields, and by the Robertson-Walker metric representation of a model universe. The concept of the cosmological substratum lends a new significance to each of these approaches and derives from them a firm theoretical basis which complements its observational basis from the astronomica discoveries of this century.

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